Optimization and Control, Assignment 2

Introduction

- Answer the questions in this assignment concerning quadratic programming (1), sequential quadratic programming (2), nonlinear programming (3), and optimization of dynamical systems (4).
- You need **80 points** or more to pass the assignment. The maximum is 110 points.
- upload your solutions or email them to `agha@mmmi.sdu.dk`.
- the deadline is **May 1, 23:59 CEST**, and
- remember that submission after the deadline is not possible.

1 Quadratic Programming (30 points)

1.1 Minimum point (10 points)

Solve exercises 16.1 on page 492 from [NW06].

1.2 Minimum distance (10 points + Bonus 10 points)

Solve exercises 16.2 on page 493 from [NW06]. Bonus 10 points if you are able to show the shortest distance from x_0 to the solution set of $Ax = b$ is $|b - Ax_0|/||A||_2$, if A is a row vector.

2 Sequential Quadratic Programming (20 points)

When developing a local SQP method, we approximate the NLP

$$\begin{aligned} \min_x \quad & f(x) \\ \text{s.t.} \quad & c(x) = 0 \end{aligned} \tag{1}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ and $c : \mathbb{R}^n \to \mathbb{R}^m$ are smooth functions, as the QP

$$\begin{aligned} \min_p \quad & f_k + \nabla f_k^\top p + \frac{1}{2} p^\top \nabla_{xx}^2 \mathcal{L}_k p \\ \text{s.t.} \quad & A_k p + c_k = 0 \end{aligned} \tag{2}$$

at the iterate (x_k, λ_k). See Section 18.1 in the textbook.

- Explain how we arrive at the approximation and derive both the objective function and the constraint in the quadratic program.
- State the KKT conditions for the quadratic program as a matrix equation.

3 Nonlinear Programming (20 points)

3.1 KKT condition (10 points)

Consider the following minimization problem

$$\begin{aligned} \min_{x} \quad & c_1 x_1 - 4x_2 - 2x_3 \\ \text{s.t.} \quad & x_1^2 + x_2^2 \leq 2 \\ & x_1^2 + x_3^2 \leq 2 \\ & x_2^2 + x_3^2 \leq 2 \end{aligned} \tag{3}$$

where c_1 is a constant.

- Decide whether it is a convex optimization problem or not.
- Write down the KKT-conditions for the problem.
- Are there any values for the constant c_1, which make the point $x = (1.4, 0.2, 0.2)^\intercal$ an optimal solution to the problem?

3.2 Minimum points (10 points)

Let $f(x) = x_1^2 x_2^4 x_3^6$, where $x = (x_1, x_2, x_3)^\intercal \in \mathbb{R}^3$.

- Determine whether $x = (0, 0, 0)^\intercal$ is a *global* optimal solution to the problem to minimize $f(x)$ under the constraint $x_1^2 + x_2^2 + x_3^2 \leq 1$.
- Determine whether $x = (0, 0, 0)^\intercal$ is a *local* optimal solution to the problem to minimize $f(x)$ under the constraint $x_1^2 + x_2^2 + x_3^2 \leq 1$.
- Determine whether f is a convex function or $\mathbb{R}^3$ or not.

4 Optimization of Dynamical Systems (40 points)

Implement open loop control for cruise control system described here: `http://ctms.engin.umich.edu/CTMS/index.php?example=CruiseControl&section=SystemModeling`, with time horizon = 10 and reference point = 20 m/s. Hints can be found in Section 3.5 [FH13].

References

[FH13] Bjarne Foss and Tor Aksel Heirung. *Merging Optimization and Control.* NTNU, 2013.

[NW06] Jorge Nocedal and Stephen Wright. *Numerical optimization.* Springer Science & Business Media, 2006.